HOMEWORK SET 5
SPRING 2021

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* Due Monday May 24, 2021.

1. Denote by G be an abelian group freely generated α and β. Endow $\mathbb{R}^3$ with the standard cubulation with vertices $\mathbb{Z}^3$. Suppose G acts on $\mathbb{R}^3$ by translations $\alpha(x, y, z) = (x+1, y+1, z)$ and $\beta(x, y, z) = (x, y+1, z+1)$. Is the quotient cube complex $X = \mathbb{R}^3/G$ special? How many hyperplanes are there in X?

2. Suppose that X and Y are special cube complexes. Show that $X \times Y$ is also a special cube complex.

3. Suppose that X is a special cube complex. Show that any covering space of X is also a special cube complex.

4. Suppose that X is a special cube complex. Does every hyperplane of X inherit a special cube complex structure (as cubulated with the midcubes)?

5. Show that the fundamental group of an orientable closed surface of genus 2 embeds into a finitely generated right-angled Artin group.