HOMEWORK SET 4
SPRING 2021

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* *Due Monday May 10, 2021. For reference of special cube complexes, see:*

[HW] F. Haglund and D. Wise, *Special cube complexes*, Geom. Funct. Anal. 17 (2008), no. 5, 1551–1620.

1. Let S be an orientable closed surface of genus two with a presentation of its fundamental group $(a_1, b_1, a_2, b_2; [a_1, b_1][a_2, b_2])$ as usual. (We adopt the notation $[a, b] = aba^{-1}b^{-1}$ for a commutator.) Construct an explicit finite quotient of $\pi_1(S)$ separating $c = [a_1, b_1]$ from the subgroup $\langle a_1, a_2 \rangle$.

2. Let $G = F_1 \times \cdots \times F_m$ be a direct product of finitely generated free groups. Show that G is isomorphic to a right-angled Artin group.

3. Let Γ be a cyclic graph of $n \geq 3$ vertices, (so the vertices are v_i and the edges are $[v_i, v_{i+1}]$, for all $i \in \mathbb{Z}/n\mathbb{Z}$. Compute the Euler characteristic of the right-angle Artin group associated to Γ, (that is, the Euler characteristic of the Salvetti complex $\mathrm{ART}(\Gamma)$ as in [Section 2.3, HW]).

4. Let X and Y be (connected) finite-dimensional cube complexes. If Y is nonpositively curved and if $f \colon X \to Y$ is a local isometry, show that X is nonpositively curved and that f is π_1–injective. (*Hint*: See [Lemma 2.11, HW]. Fill in details reflecting your comprehension.)

5. Are right-angled Artin groups all torsion-free? Justify your answer.