HOMEWORK SET 3
SPRING 2021

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* *Due Monday April 26, 2021.*

1. Recall that a subgroup H is said to be *separable* in a group G, if for every $g \in G$ not in H, there exists a finite quotient $\bar{G}$ of G, such that the image $\bar{g}$ is not in $\bar{H}$. Suppose that M is an orientable closed 3–manifold which fibers over a circle with a surface fiber S. Show that the subgroup $\pi_1(S)$ is separable in $\pi_1(M)$.

2. Let M be a closed orientable hyperbolic 3–manifold. Show that $\pi_1(M)$ does not contain any free abelian subgroup of rank 2. (Hint: Classify all discrete abelian subgroups in $\mathrm{PSL}(2,\mathbb{C})$.)

3. Let M be a closed orientable hyperbolic 3–manifold. Assume the fact that $\pi_1(M)$ is residually finite. Show that for any $R > 0$, there exists a finite cover M' of M, such that the injectivity radius of M' is everywhere greater than R.

4. Prove that any finitely generated subgroup of $\mathrm{PSL}(2,\mathbb{C})$ is residually finite. (Hint: Look up the keyword "Selberg lemma" online, or in Ratcliffe's book *Foundations of Hyperbolic Manifolds*.)

5. Let G be a finitely generated word hyperbolic group. Is every finitely generated subgroup of G separable? Do literature search, and decide if the above question has an affirmative answer, or has a counter example, or has no conclusive answer yet.