HOMEWORK SET 2
SPRING 2021

INSTRUCTOR: YI LIU

* Due Monday April 12, 2021.

1. Up to homeomorphism, classify all the compact 3-manifolds that are homeomorphic to a compact interval bundle over a Klein bottle.

2. Let M be an orientable connected closed irreducible 3–manifold. Suppose that M contains no JSJ piece that is homeomorphic to the thickened Klein bottle (that is, the compact interval bundle over a Klein bottle whose bundle space is orientable). Let $\mathcal{T}$ be a union of tori that makes a JSJ decomposition of M. Let M' be a finite cover of M. Show that the preimage $\mathcal{T}'$ of $\mathcal{T}$ in M' is a union of tori that makes a JSJ decomposition of M'.

3. Let M be an orientable connected closed irreducible 3–manifold. Show that there exists a finite cover of M that contains no embedded Klein bottles.

4. In hyperbolic plane $\mathbb{H}^2$, let Δ be the interior of an ideal triangle. Denote by a, b, c the sides of Δ, which are geodesics in $\mathbb{H}^2$. Denote by $R_a, R_b, R_c \in \mathrm{Isom}(\mathbb{H}^2)$ the reflections about a, b, c, respectively. Show that R_a, R_b, R_c generate a discrete subgroup G of $\mathrm{Isom}(\mathbb{H}^2)$. Can you find a presentation of the group G?

5. Find at least five research papers online related to the Mostow rigidity theorem. Describe their main results, and explain in what sense you think they are related to that topic. Create a reference list in a typical style as you have seen in a math paper.