1. Construct a complete hyperbolic 3-manifold by gluing up the sides of a regular ideal tetrahedron. The manifold $M$ is called the Gieseking manifold. Show that the link of the cusp point of $M$ is a Klein bottle.

2. Suppose that $M$ is an orientable closed hyperbolic 3-manifold. Show that $\pi_1(M)$ does not contain a subgroup isomorphic to $\mathbb{Z} \times \mathbb{Z}$. (Hint: Maybe you need to show first that any such subgroup would be parabolic.)

3. A horocusp $V = B/\Gamma$ is the quotient of a horoball $B \subset \mathbb{H}^3$ by a discrete rank-two parabolic subgroup $\Gamma$ which stabilizes $B$. (So $V$ is homeomorphic to $T^2 \times [0, +\infty)$.) Show that $\text{Area}_{\mathbb{H}}(\partial V) = \text{Vol}_{\mathbb{H}}(V)$.

4. We consider a 2–dimensional concrete example to understand completeness. In the Poincaré disk model, consider the convex polygon $R$ with ideal vertices $i, u, -i, -u$, where $u = e^{i\theta}$ for some $0 \leq \theta < \pi/2$. Suppose that $f, g$ are the hyperbolic translations such that $f(i) = u, f(-u) = -i, g(-i) = u, g(-u) = i$. Moreover, suppose the axes of $f$ and $g$ are the common orthogonals of the pairs of the opposite edges of $R$, respectively. Then we obtain a hyperbolic surface $S$ from $R$ by the side pairing maps $f$ and $g$. Decide if the resulting hyperbolic surface $S$ is complete.