HOMEWORK SET 3
SPRING 2020

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* *Due Thursday April 2, 2020.*

1. Construct a complete hyperbolic 3-manifold by gluing up the sides of a regular ideal tetrahedron. The manifold M is called the *Gieseking manifold*. Show that the link of the cusp point of M is a Klein bottle.

2 Suppose that M is an orientable closed hyperbolic 3-manifold. Show that $\pi_1(M)$ does not contain a subgroup isomorphic to $\mathbb{Z} \times \mathbb{Z}$. (*Hint*: Maybe you need to show first that any such subgroup would be parabolic.)

3. A *horocusp* $V = B/\Gamma$ is the quotient of a horoball $B \subset \mathbb{H}^3$ by a discrete rank-two parabolic subgroup Γ which stabilizes B. (So V is homeomorphic to $T^2 \times [0, +\infty)$.) Show that $\mathrm{Area}_{\mathbb{H}}(\partial V) = \mathrm{Vol}_{\mathbb{H}}(V)$.

4. We consider a 2–dimensional concrete example to understand completeness. In the Poincaré disk model, consider the convex polygon R with ideal vertices $i, u, -i, -u$, where $u = e^{i\theta}$ for some $0 \leq \theta < \pi/2$. Suppose that f, g are the hyperbolic translations such that $f(i) = u, f(-u) = -i, g(-i) = u, g(-u) = i$. Moreover, suppose the axes of f and g are the common orthogonals of the pairs of the opposite edges of R, respectively. Then we obtain a hyperbolic surface S from R by the side pairing maps f and g. Decide if the resulting hyperbolic surface S is complete.