HOMEWORK SET 7
SPRING 2017

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* *Due Wednesday May 31, 2017.*

1. Let M be a closed manifold (or a topological space in general). Verify that on any dimension k, the Gromov norm $\|\cdot\|\colon H_k(M;\mathbb{R}) \to [0,+\infty)$ is a semi-norm (for real linear spaces), namely, such that $\|u\alpha\| = |u|\|\alpha\|$ and $\|\alpha+\beta\| \leq \|\alpha\| + \|\beta\|$ hold true for all $u \in \mathbb{R}$ and $\alpha, \beta \in H_k(M;\mathbb{R})$.

2. Show that spheres and (orientable) projective spaces have vanishing simplicial volume.

3. Suppose M and N are orientable closed manifolds of dimension m and n respectively. Show that there exists a constant $C = C(m,n) > 0$ such that $\|M \times N\| \leq C\,\|M\| \times \|N\|$. (*Remark*: It is also true that $C'\|M\| \times \|N\| \leq \|M \times N\|$. You can find a proof of the latter fact in Gromov's paper "Volume and bounded cohomology".)