HOMEWORK SET 6
SPRING 2017

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* *Due Wednesday May 17, 2016.*

1. For complete closed hyperbolic 3-manifolds, state the Mostow Rigidity in geometric terms and group-theoretic terms. Verify that they are equivalent.

2. Let $U_1, U_1', \cdots, U_k, U_k'$ be $2k$ half-spaces (bounded by hyperplanes) in $\mathbb{H}^3$ such that their closures in $\mathbb{H}^3 \cup \partial_\infty \mathbb{H}^3$ with respect to the natural compactification are pairwise disjoint. Suppose that g_i is an orientation-preserving isometric transformation that takes $\mathbb{H}^3 \setminus U_i$ to U_i, for each $i = 1, \cdots, k$. The subgroup $\Gamma = \langle g_1, \cdots, g_k \rangle$ of $\mathrm{Isom}_+(\mathbb{H}^3)$ arising from this construction is called a (3-dimensional) *Schottky group* of rank k. Show that Γ is isomorphic to a free group of rank k. Does it satisfy the conclusion of the Mostow rigidity? Justify your answer.

3. Let Γ be a Schottky group of rank k. Show that $\mathbb{H}^3/\Gamma$ is homeomorphic with the interior of a compact handle-body with k handles. (*Hint*: You may want to decompose $\partial_\infty \mathbb{H}^3$ into the *limit set* $\Lambda(\Gamma)$ and the *domain of discontinuity* $\Omega(\Gamma)$ with respect to the action of Γ. Learn these terms from available sources if you haven't seen them before. Then $(\mathbb{H}^3 \cup \Omega(\Gamma))/\Gamma$ will be a compact handle-body with k handles.)