HOMEWORK SET 6
SPRING 2016

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* Due Tuesday May 3, 2016.

1. Use Margulis' Lemma to conclude that for sufficiently small $\epsilon > 0$, the ϵ-thin part of a hyperbolic 3-manifold has one of the three possible shapes: a solid torus, or a rank-1 cusp, or a rank-2 cusp.

2. Show that if a torsion-free subgroup of $\mathrm{PSL}_2(\mathbb{C})$ has nontrivial center, it must be abelian. In particular, virtually nilpotent subgroups of $\mathrm{PSL}_2(\mathbb{C})$ are virtually abelian.

3. Let G be a finitely generated group with generators $x_1, \cdots, x_n$ and relators $r_1, \cdots, r_m$ which are words of the generators. The set $R(G)$ of $\mathrm{SL}_2(\mathbb{C})$–representations of G can be identified as $f^{-1}(I, \cdots, I)$ of the map:
$$f : \mathrm{SL}_2(\mathbb{C})^{\times n} \to \mathrm{SL}_2(\mathbb{C})^{\times m}$$
defined to be $f(A_1, \cdots, A_n) = (r_1(A_1, \cdots, A_n), \cdots, r_m(A_1, \cdots, A_n))$. This endows $R(G)$ with a topology induced by the (Lie group) topology of $\mathrm{SL}_2(\mathbb{C})$.
   (1) Verify that the induced topology of $R(G)$ does not depend on the choice of the generators and relators.
   (2) For any homomorphism $\phi : G \to H$ between finitely generated groups, show that there is an induced continuous map $\phi^* : R(H) \to R(G)$. It is an embedding if ϕ is surjective.
   (3) Identify $R(G * H)$ with $R(G) \times R(H)$.