HOMEWORK SET 5
SPRING 2017

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* *Due Wednesday May 3, 2016.*

1. A discrete subgroup of $\mathrm{PSL}_2(\mathbb{C})$ is said to be *elementary* if it has a finite orbit in $\mathbb{H}^3 \cup \hat{\mathbb{C}}$. If Γ is a torsion-free discrete subgroup of $\mathrm{PSL}_2(\mathbb{C})$ without parabolic elements, show that every nontrivial elementary subgroup of Γ is infinite cyclic. (*Hint*: Show that any such subgroup must preserve a common geodesic.)

2. Suppose that M is a compact orientable 3-manifold with at least one boundary component of genus > 1. Show that the interior of M does not admit a complete hyperbolic structure of finite volume.

3. A sequence of discrete subgroups $(\Gamma_n)_{n\in\mathbb{N}}$ of $\mathrm{PSL}_2(\mathbb{C})$ is said to *converge geometrically* to a discrete subgroup Γ_∞, if every $g_\infty \in \Gamma_\infty$ is the limit of some sequence $(g_n \in \Gamma_n)_{n\in\mathbb{N}}$. If Γ_n are infinite cyclic, is it true that Γ_∞ is infinite cyclic?