

HOMEWORK SET 5
SPRING 2016

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** Due Tuesday May 3, 2016.*

1. If a sequence of discrete subgroups $\{\Gamma_n\}_{n \in \mathbb{N}}$ of $\mathrm{PSL}_2(\mathbb{C})$ converge geometrically to a discrete subgroup Γ_∞ and Γ_n are torsion-free, is it true that Γ_∞ is torsion-free?
2. Let M be a hyperbolic 3-manifold containing a closed geodesic γ . Show that if the length l of γ is sufficiently short, then γ is embedded, so for some radius $r = r(l)$, the r -neighborhood $V_r(\gamma)$ of γ is an isometrically embedded solid torus. Moreover, show that $r(l)$ can be chosen so that $r(l) \rightarrow +\infty$ as $l \rightarrow 0+$. In other words, short geodesics have deep tube neighborhood.
3. Suppose that M is a compact orientable 3-manifold with at least one boundary component of genus > 1 . Show that the interior of M does not admit a complete hyperbolic structure of finite volume.
4. For every value $V > 0$, show that there are at most finitely many complete hyperbolic 3-manifolds of volume equal to V , up to isometry.