HOMEWORK SET 4
SPRING 2016

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* *Due Tuesday April 19, 2016.*

1. Let M be an orientable compact 3-manifold with boundary a finite collection of tori $P_1, \cdots, P_m$. A *Dehn filling* N of M is given by gluing M and a collection of solid tori $V_i = S^1 \times D^2$ by homeomorphisms $P_i \cong \partial V_i$. Denote by $s_i \subset P_i$ be the simple closed curve given by $* \times \partial D^2$ via the homeomorphism.

    (1) Show that the homeomorphism type of N is determined by the isotopy class of c_i on P_i. In other words, a Dehn filling is determined by the slope that it fills.

    (2) Show that if d_i is any simple closed curve of T_i that intersects c_1 exactly once, then d_i is homotopic to the core curve $S_1 \times *$ of V_i in N.

2. Use the gluing equation and the holonomy of the vertex link to show the following statement. For any given $\epsilon > 0$, there are only finitely many hyperbolic Dehn fillings of the figure-eight knot complement such that the length of (the unique geodesic representative of) the core curve is greater than ϵ. In other words, most hyperbolic Dehn fillings have a short core.

3. If T is an ideal hyperbolic tetrahedron with dihedral angles α, β, γ, then Milnor's formula asserts

$$\mathrm{Vol}_{\mathbb{H}}(T) = \Lambda(\alpha) + \Lambda(\beta) + \Lambda(\gamma)$$

where

$$\Lambda(\theta) = -\int_0^\theta \log|2\sin u|\,\mathrm{d}u$$

is the Lobachevsky function, (see Chapter 7 of Thurston's notes). Use this fact to conclude that the maximum volume of ideal hyperbolic tetrahedra is achieved by the regular ideal tetrahedron. It follows that the complete hyperbolic structure of the figure-eight complement is greater than any of its hyperbolic Dehn fillings.