HOMEWORK SET 3
SPRING 2016

INSTRUCTOR: YI LIU

* *Due Tuesday April 5, 2016.*

1. Construct a complete hyperbolic 3-manifold by gluing up the sides of a regular ideal tetrahedron. The manifold M is called the *Gieseking manifold*. Show that the link of the cusp point of M is a Klein bottle.

2 Suppose that M is an orientable closed hyperbolic 3-manifold. Show that $\pi_1(M)$ does not contain a subgroup isomorphic to $\mathbb{Z} \times \mathbb{Z}$. (*Hint*: Maybe you need to show first that any such subgroup would be parabolic.)

3. For any ideal tetrahedron T of $\mathbb{H}^3$, show that the dihedral angles at opposite edges are equal to each other.

4. A *horocusp* $V = B/\Gamma$ is the quotient of a horoball $B \subset \mathbb{H}^3$ by a discrete rank-two parabolic subgroup Γ which stabilizes B. (So V is homeomorphic to $T^2 \times [0, +\infty)$.) Show that $\mathrm{Area}_{\mathbb{H}}(\partial V) = \mathrm{Vol}_{\mathbb{H}}(V)$.